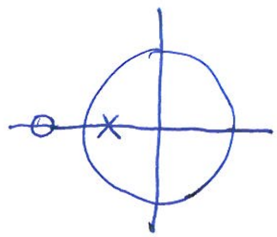
Problem 8·6

(a) $$H(z) = \frac{0.8z+1}{z+0.8}$$

(b)



zero: -1.25

pole: -0.8

(c) $$H(e^{j\omega}) = \frac{0.8e^{j\omega}+1}{e^{j\omega}+0.8}$$

(d) $$|H(e^{j\omega})|^2 = H(e^{j\omega}) \cdot H^*(e^{-j\omega})$$

$$= \left(\frac{0.8e^{j\omega}+1}{e^{j\omega}+0.8}\right)\left(\frac{0.8e^{-j\omega}+1}{e^{-j\omega}+0.8}\right)$$

$\rightarrow$ num + denom are same

$\rightarrow |H(e^{j\omega})|^2 = 1$

Problem 8.8

$$y[n] = -0.9\,y[n-6] + x[n]$$

(a) $$Y(z) = -0.9 z^{-6} Y(z) + X(z)$$

$$H(z) = \frac{1}{1+0.9z^{-6}} = \frac{z^6}{z^6+0.9}$$

SIX ZEROS AT $z=0$

(b) Poles are found as the solutions to

$$z^6 + 0.9 = 0$$

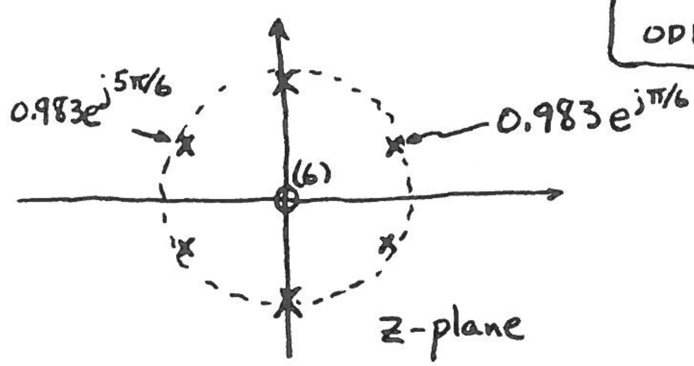
This involves the "roots of unity"

$$z^6 = -0.9 = 0.9 e^{j\pi} e^{j2\pi\ell} \qquad \ell = 0,1,2,3,4,5$$

$$z = \sqrt[6]{0.9}\, e^{j\pi/6} e^{j\pi\ell/3}$$

$$= 0.983\, e^{j\pi(2\ell+1)/6}$$

ANGLES ARE:
$\frac{\pi}{6}, \frac{3\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{9\pi}{6}, \frac{11\pi}{6}$
ODD MULTIPLES of 30°



Problem 8.12

$$y[n] = \tfrac{1}{2} y[n-1] + x[n] \quad \Longrightarrow \quad H(z) = \frac{1}{1-\frac{1}{2}z^{-1}}$$

(a) Input is $u[n] \Rightarrow X(z) = \dfrac{1}{1-z^{-1}}$

$$Y(z) = H(z)X(z) = \frac{1}{(1-z^{-1})(1-\frac{1}{2}z^{-1})}$$

Do a partial fraction expansion:

$$Y(z) = \frac{K_1}{1-z^{-1}} + \frac{K_2}{1-\frac{1}{2}z^{-1}}$$

$$K_1 = \left.\frac{1}{1-\frac{1}{2}z^{-1}}\right|_{z=1} = 2$$

$$K_2 = \left.\frac{1}{1-z^{-1}}\right|_{z=\frac{1}{2}} = \frac{1}{1-2} = -1$$

$$= \frac{2}{1-z^{-1}} - \frac{1}{1-\frac{1}{2}z^{-1}}$$

$$y[n] = 2u[n] - (\tfrac{1}{2})^n u[n]$$

(b) $x[n] = e^{j(\pi/4)n}u[n] \Rightarrow X(z) = \dfrac{1}{1-e^{j\pi/4}z^{-1}}$

$$Y(z) = H(z)X(z) = \frac{1}{(1-\frac{1}{2}z^{-1})(1-e^{j\pi/4}z^{-1})}$$

Partial Fraction Expansion:

$$Y(z) = \frac{A_1}{1-\frac{1}{2}z^{-1}} + \frac{A_2}{1-e^{j\pi/4}z^{-1}}$$

$$A_1 = \left.\frac{1}{1-e^{j\pi/4}z^{-1}}\right|_{z=1/2} \cong 0.68e^{j.59\pi}$$

$$A_2 = \left.\frac{1}{1-\frac{1}{2}z^{-1}}\right|_{z=e^{j\pi/4}} \cong 1.36e^{-j.16\pi}$$

$$y[n] = \underbrace{0.68e^{j0.59\pi}(\tfrac{1}{2})^n u[n]}_{\text{This term dies out}} + \underbrace{1.36e^{-j0.16\pi}(e^{j\pi n/4})u[n]}_{\text{Steady-state term}}.$$

(c) $\mathcal{H}(\hat{\omega})\big|_{\hat{\omega}=\pi/4} = H(e^{j\pi/4}) = H(z)\big|_{z=e^{j\pi/4}}$

$$H(e^{j\pi/4}) = \frac{1}{1-\frac{1}{2}e^{-j\pi/4}} \cong 1.36e^{-j0.16\pi}$$

which is the same as A_2

Problem 8.16

General comments:

(1) Pole-Zero plots # 1 & #2 are for FIR filters because all poles are at $z=0$

(2) Freq. Responses A & C have many zeros which correspond to zeros on UNIT CIRCLE

(3) Freq Resp C is a Lowpass filter so its pole-zero plot should have no zero near $z=0$

$\therefore$ PZ #1 $\longleftrightarrow$ C

#2 $\longleftrightarrow$ A

(4) Freq. Resp. D has prominent peaks at $\pm\pi/2 \Rightarrow$ poles near $z = e^{\pm j\pi/2} = \pm j$.

$\therefore$ #6 $\longleftrightarrow$ D.

(5) Freq Resp. E has several peaks each caused by different pole-pairs. Notice peaks at $\hat{\omega} \simeq \pm\pi/6$.

$\therefore$ #3 $\longleftrightarrow$ E

(6) Freq. Response B is Highpass filter which needs a pole near $z=-1$

$\therefore$ #5 $\longleftrightarrow$ B.