EE 477
Digital Signal Processing

7b

z-Transforms

Inverse and Deconvolution

- We have seen that two systems in cascade can be combined into a single system by multiplying $H_1(z)$ and $H_2(z)$.
- We can also take a system function H(z) and *factor* it into two or more low-order systems.
- Question: can we *divide* the system output by the system function (“deconvolve”) and recover the input?

$$Y(z) = H_1(z)X(z); \quad Y(z)H_2(z) \stackrel{?}{=} X(z)$$

$$Y(z) = H_1(z)H_2(z)Y(z) \Rightarrow H_1(z)H_2(z) = 1?$$

Inverse and Deconvolve, cont.

- If we can find $H_2(z)$, it is called the *inverse* of $H_1(z)$.
- NOTE that $H_2(z)$ will not be FIR if $H_1(z)$ is FIR.
- $H_2(z)$ may represent a non-causal and/or unstable system even if $H_1(z)$ is causal and stable.

Relating H(z) and H($e^{j\omega}$)

- NOTE CAREFULLY: z-transform and frequency response formulae are of identical form.

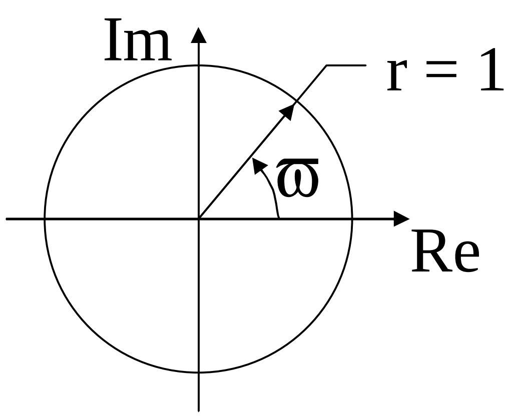
$$H(\hat{\omega}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k} \qquad H(z) = \sum_{k=0}^{M} b_k z^{-k}$$

- If we evaluate H(z) for z=$e^{j\omega}$, it is clear:

$$H(\hat{\omega}) = H(e^{j\hat{\omega}}) = H(z)\Big|_{z=e^{j\omega}}$$

Properties of $z=e^{j\varpi}$

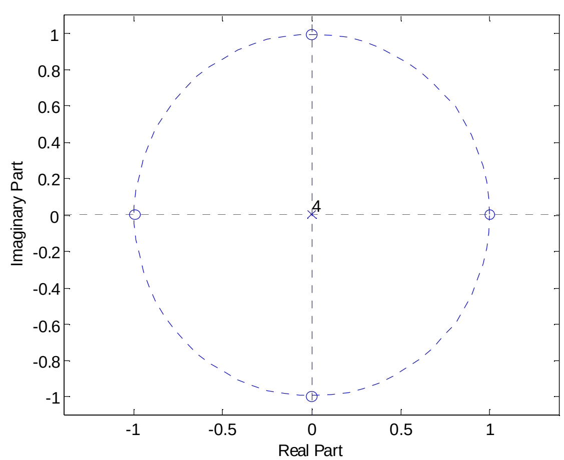
- Observe $z=e^{j\varpi}$ for $-\pi< \varpi <\pi$: $|z|=1$, phase= ϖ
- This defines a *circle* in the z-plane with radius=1: referred to as the *unit circle*

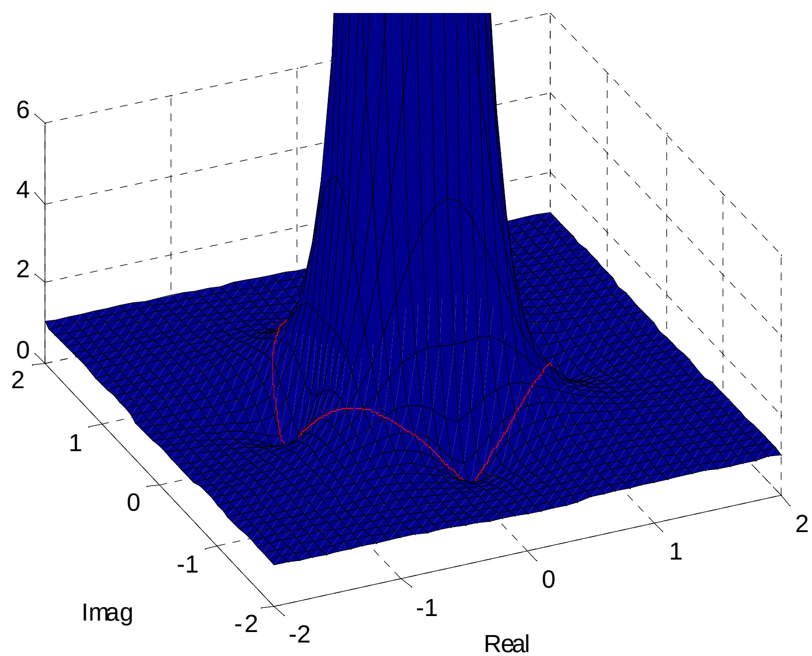


Visualizing Frequency Response

- We can observe z-transform along the unit circle to reveal the frequency response.

$$H(z) = 1 - z^{-4}$$





Poles and Zeros

- A *pole* in the z-domain is a value of *z* that "pushes up" the magnitude like a tent pole.
- A *zero* in the z-domain is a value of *z* that "pins down" the magnitude like a stake or tack.
- The pole and zero locations control the magnitude everywhere, *including along the unit circle*.

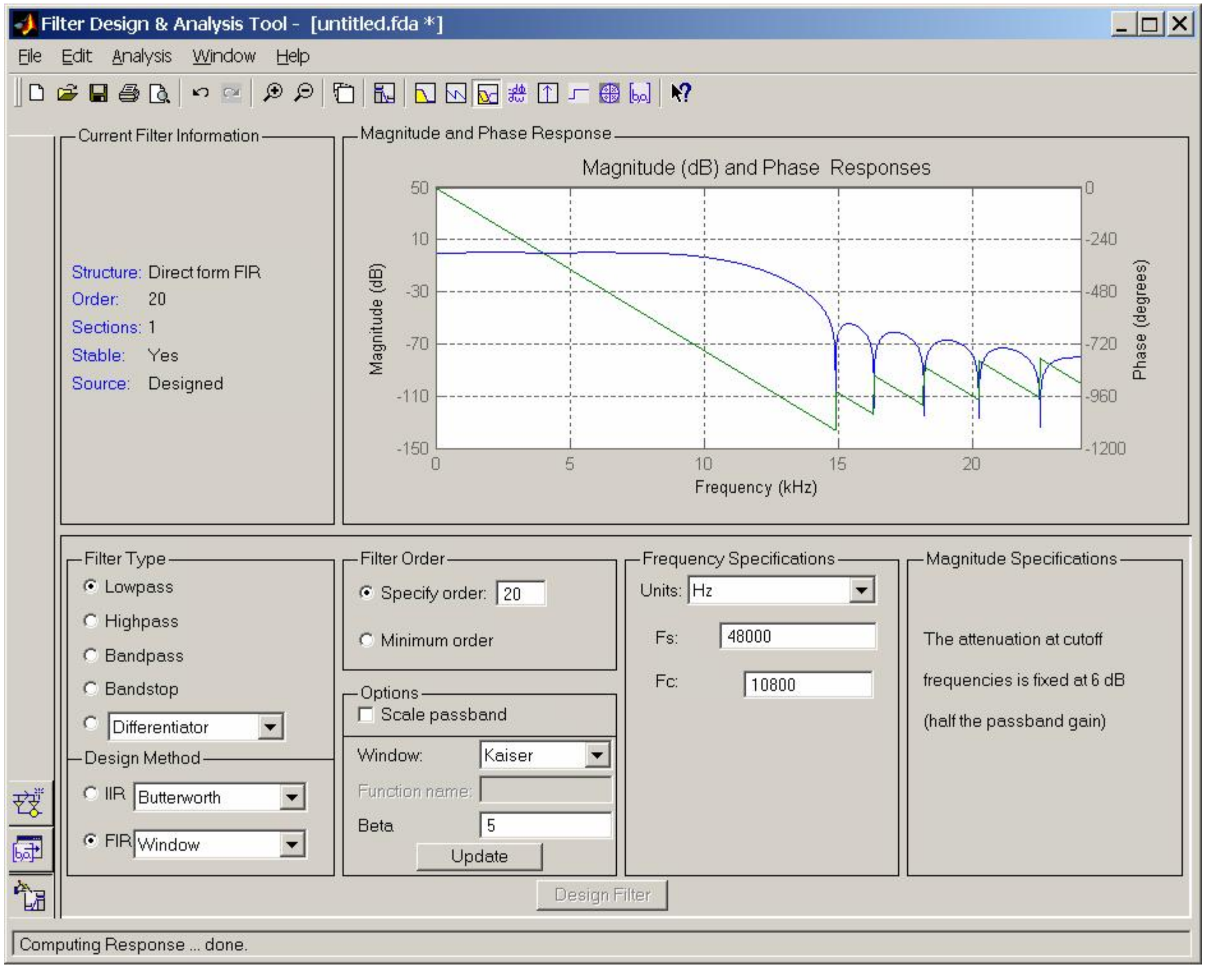
FIR Systems

- FIR systems contain only finite zeros. Poles are located at zero (and perhaps infinity).
- FIR filter design requires a careful choice of zero locations.
- Stop band has zeros on the unit circle.
- Pass band has zeros off the unit circle.

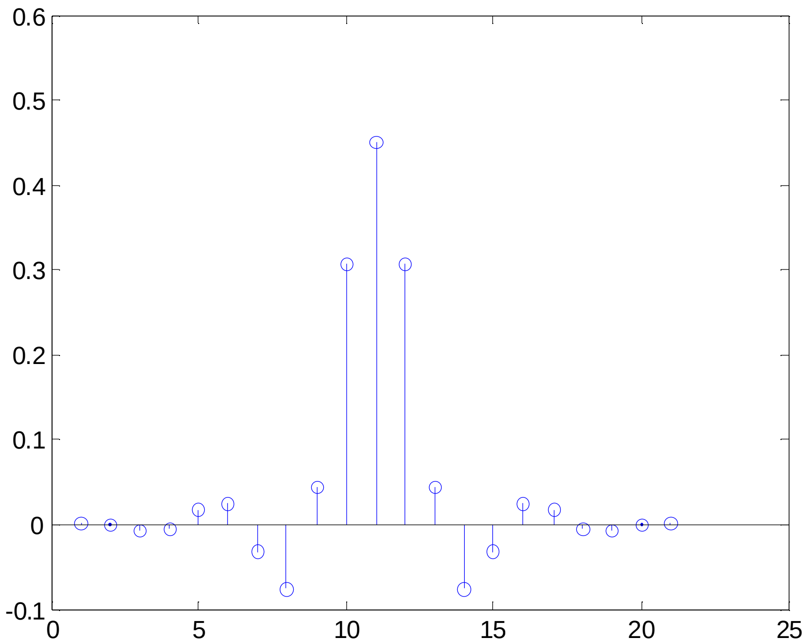
Matlab FIR Filter Design

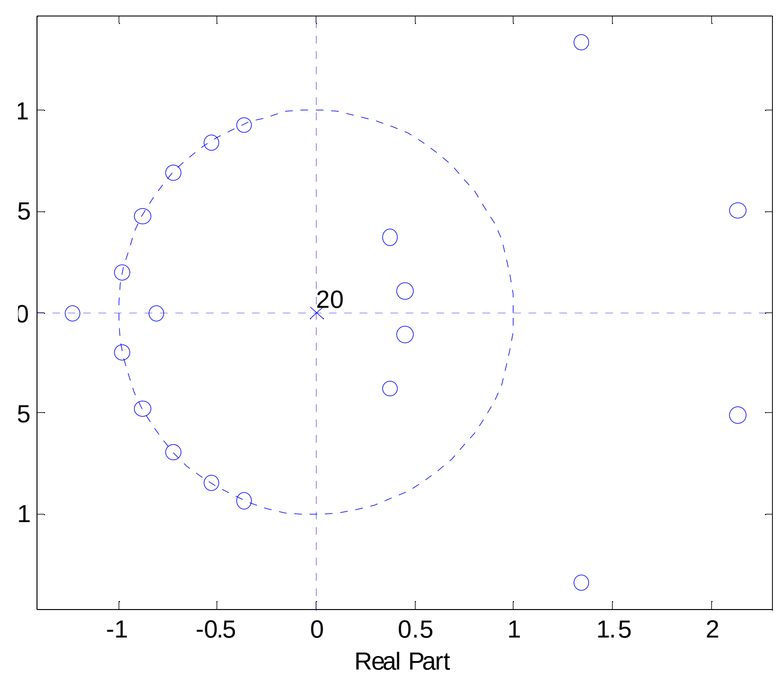
- Matlab provides several FIR filter design tools, including: `fir1`, `fir2`, and `remez`
- Matlab GUI: Filter Design and Analysis Tool (FDATool)
- Usually specify passband ripple, stopband attenuation, band edges, filter order, and f_s

Design Example



Design Example (cont.)





Symmetry and Linear Phase

- FIR systems with symmetric coefficients ($b_k = b_{M-k}$) have frequency responses with *linear phase*.
- Show this by grouping z-transform terms, for example:

$$H(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_2 z^{-4} + b_1 z^{-5} + b_0 z^{-6}$$
$$= z^{-3}\left[b_0\left(z^3 + z^{-3}\right) + b_1\left(z^2 + z^{-2}\right) + b_2\left(z^1 + z^{-1}\right) + b_3\right]$$

Linear Phase (cont.)

- Now evaluate H(z) on unit circle:

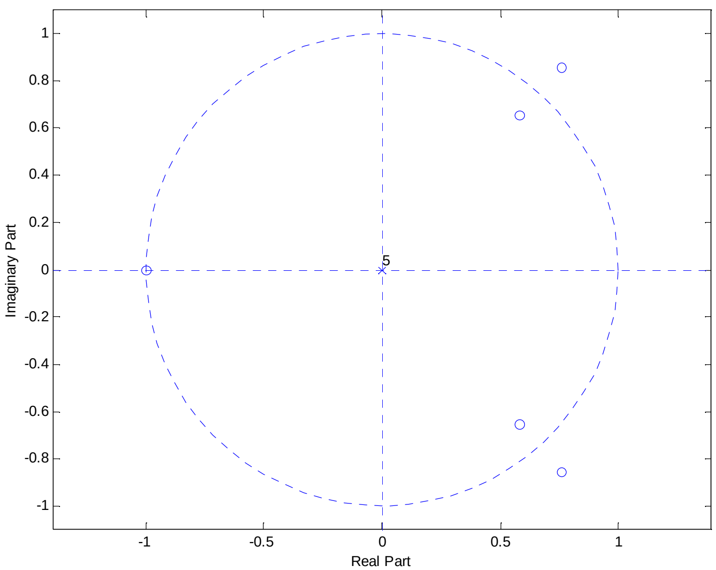
$$H(e^{j\hat{\omega}}) = \underbrace{e^{-j3\hat{\omega}}}_{\substack{\text{linear}\\ \text{phase}\\ \text{term}}} \underbrace{\left[b_0 \underbrace{\left(e^{j3\hat{\omega}} + e^{-j3\hat{\omega}}\right)}_{2\cos(3\hat{\omega})} + b_1 \underbrace{\left(e^{j2\hat{\omega}} + e^{-j2\hat{\omega}}\right)}_{2\cos(2\hat{\omega})} + b_2 \underbrace{\left(e^{j\hat{\omega}} + e^{-j\hat{\omega}}\right)}_{2\cos(\hat{\omega})} + b_3 \right]}_{\text{a real function of } \hat{\omega} \text{ (phase=0)}}$$

- Example if M is odd:

$$H(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + b_2 z^{-3} + b_1 z^{-4} + b_0 z^{-5}$$
$$= z^{-2.5}\left[b_0\left(z^{2.5} + z^{-2.5}\right) + b_1\left(z^{1.5} + z^{-1.5}\right) + b_2\left(z^{0.5} + z^{-0.5}\right)\right]$$

Zero Symmetry

- For an FIR linear phase system (implies coefficient symmetry), the zeros will have a specific pattern. For each z_0, there will be:



$$\left\{ z_0, z_0^*, \frac{1}{z_0}, \frac{1}{z_0^*} \right\}$$